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When and How to Appropriately Implement Teaching Tools and Strategies for Early Childhood Numeracy

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Augustana College's Partnership with Longfellow Elementary:

The Education program at Augustana College has established a partnership with Longfellow Liberal Arts, a neighboring elementary school that fosters the education for children in preschool through sixth grade. This partnership has provided undergraduate elementary education majors with experiences in the classrooms throughout their junior year. Augustana College strives to nurture professionalism within their teacher candidates and prepare them for both the moments that remind us that teaching can be an aspiration rather than an occupation, as well as the troubling moments that may present themselves to a beginning teacher. The partnership between Augustana College and Longfellow Elementary has provided teacher candidates with hands-on experiences working with students one-on-one, in a small group setting, and even through whole-class instruction. The methods courses taught at Augustana have provided us with the opportunity to work with multiple age groups on multiple concepts and skills. We have come to get to know many of the students at Longfellow Elementary through these experiences and are grateful for the variety of opportunities Augustana College has provided for us.

Though Augustana College had established a partnership with Longfellow six years ago, we were first included in this collaboration in August 2014. All teacher candidates are required to enroll in Mathematical Methods, instructed by Dr. Mike Egan, in the fall of their junior year. Math Methods is a course that is focused on learning to exercise appropriate mathematical habits of the mind, teaching elementary mathematics curricular and standards, and provides practice in building optimal mathematical learning experiences for students. Through this course, we have developed a deeper appreciation for the various approaches to problem solving and learning styles that may differ amongst students. We also applied our understanding of the current mathematical curriculum in practicing the creation of lessons that both align with the standards and provide an optimal learning experience for students. Egan has utilized the partnership with Longfellow throughout this course by pairing his students with Kindergarten students. The teacher candidates were divided into pairs or small groups and were assigned a group of three or four Kindergarteners to work with throughout the course. The Kindergarten groups were established based on similarities in their mathematical understanding and development, which was determined by the "Education Software for Guiding Instruction" (ESGI) Assessment results. Teacher candidates were required to meet with their group on a weekly basis to strengthen and develop numeracy skills. The teacher candidates were responsible for planning appropriate lessons and activities that both built on prior knowledge and introduced new concepts and skills. Though Dr. Egan did his best to group Kindergarten students based on their existing skills, teacher candidates understood at times they needed to make provisions for individual differences and student needs within their lessons. Because we both were assigned the same group, we share similar experiences and insights. Though we only worked with our group of kindergarteners for ten weeks, we got to know our students well and developed a close bond with them. We both feel this experience provided us with ample opportunities to practice collaboration, lesson planning, differentiation, and effective instructional delivery, which in turn have enhanced our professional development as educators. We hoped by continuing this program we would grow closer to more of the Longfellow Kindergarteners and develop into stronger perspective teachers.

Just before our Math Methods course had concluded, all teacher candidates were informed by Dr. Egan and Dr. Hengst of the opportunity to continue working on developing number sense with the kindergarten students at Longfellow. We both expressed interest in this program and sought out more information of what this program would entail. We soon realized this was something we were both very interested in and decided it was ultimately the most beneficial way for us to utilize our Augie Choice grant of two thousand dollars. Thus, we embarked on this two-term project at the beginning of winter term. Participating in the Number Sense Program has allowed us to continue putting to practice the knowledge and skills we gained from our Math Methods course in the fall (September – November). The Number Sense Program was developed to provide purposeful, individualized instruction for Longfellow's kindergarteners, as well as enhance the participating teacher candidates' professional development. This program has enhanced the learning opportunities for Longfellow students because participating teacher candidates have been able to address particular needs of each child in the classroom. Teacher candidates in this program assess each student individually, plan appropriate learning experiences for each child, and analyze the results of their instructional interventions. Because it is difficult for even the most talented teacher to meet the needs of all his or her students, this program supports their efforts to target individual student needs and learning styles. It is clear that this unique program provides Longfellow Kindergarteners with enriched learning experiences they might not otherwise have. Not only does this program provide the Kindergarteners with additional practice and opportunities to develop, but it also provides the participating teacher candidates with the opportunity to practice directly instructing a group of students and develop into a professional educator. This program requires the undergraduate elementary majors to spend seven to eight hours a week planning effective lessons and further delivering the planned instruction to their students. While participating in this program, teacher candidates are responsible for pursing a research project that coincides with their work at Longfellow. They will further develop their professionalism as they present their findings at the 2015 Illinois Council of Teachers of Mathematics conference as well as at the Celebration of Student Learning on Augustana's campus in the spring of 2015. The Number Sense Program has the potential to be unique to each individual teacher candidate. Their work with the Kindergarteners at Longfellow, their research, and overall experience will vary based on their personal interests as well as how they believe they can best contribute to or enhance student learning.

We were excited to finally be accepted into the program and for the opportunity to meet with the rest of the Kindergarten students we had not worked with in the fall (September – November). After only one day, we were both willing to admit that we were taken by surprise in the large range of mathematical understanding and level of development demonstrated by the Kindergarteners. Because we had only worked with one group in which the students had demonstrated a similar level of understanding, we did not realize such a large range of understanding and ability existed. We found some students could only successfully recognize numbers one through five consistently, while other students demonstrated an understanding for more complex mathematical concepts such as comparing numbers and problem solving strategies. Thus, we knew the focus of our lesson plans would differ based on the needs of each group. With only a few weeks into the program, we had already learned and implemented a variety of teaching techniques and strategies that could be used across multiple levels of understanding and development. Throughout this program, we have spent a great deal of time finding and choosing appropriate activities and accommodations that would ultimately benefit learning for all students. It is because of experiences like these that we feel this program has further contributed to the development of our teaching. When reflecting on our interactions with our students on the first day, we realize that because we were inexperienced teachers we struggled to plan effective lessons that would benefit student learning. Compared to the interactions we have with our students now, it is quite obvious that we have grown immensely as teacher candidates. When planning our lessons, we now focus on the content and skills of the lesson, and how the students will engage themselves in it, rather than how we can make the activity "cute" and "fun". We have seen a great transition in our teaching; we have moved from a stage where we felt a lack of confidence and uncertainty in the activities we were implementing in our lessons, to a stage where we evaluate assessments and gather evidence of student learning to further plan appropriate lessons based on individual needs and learning styles. We have come to understand that this transition in our teaching ability is due to the additional experiences this program has provided us with. Thus, we are extremely grateful for Augustana College and the Education Department for providing us with the opportunity to enhance the learning of the Kindergarten students as well as the opportunity to develop into more proficient and professional teachers.

What is Number Sense?

We have researched and evaluated multiple definitions for number sense and have come to the conclusion that researchers emphasize different skills as evidence of number sense. Many researchers agree, however, that number sense is the "ability to subitize small quantities, to discern number patterns, to compare numerical magnitudes and estimate quantities, to count, and to perform simple number transformations are key elements of number sense in young children" (Jordan, Kaplan, Nabors Olah & Locuniak, 2006, 154). Other scholars have stated that number sense refers to "fluidity and flexibility with numbers" (Gersten & Chard, 1999, 18-28). Through our research and our experience in the Number Sense Program, we agree most specifically with Van De Walle's definition, in which he defines number sense as the gradual mathematical development of thinking about and working with numbers. He also believes that number sense develops as students comes to understand multiple ways to think about and represent a number, learn to utilize numbers, and develop an understanding of how to use numbers in problem-solving situations (Van De Walle, 2007, 124). In our Math Methods class, Dr. Egan provided us with a useful article, "Teaching Numbers in Early Elementary Years". This article compared number sense to Language Arts, which ultimately allowed us to expand our understanding for this complex term. Cain and Faulkner, the authors of this article, compared students' understanding for numbers to their ability to read. They explained how some children can "read, but then again not really *read".* By this, they meant the child is reading, or simply saying the word without comprehending any meaning of the text. Similarly, a child can identify a number by reading or writing the symbolic number, without having developed an "understanding of the concrete concept that this abstract orthographic symbol represents" (Cain & Faulkner, 2011, 290). For example, a student may be able to identify the symbolic number four, but may struggle with the concept that this symbolic number represents the quantity of four. Therefore, he or she is unable to “do” math. Number sense includes a wide range of concepts and skills. In 1989, the National Council of Teachers “identified the following five components that characterize number sense: number meaning, number relationships, number magnitude, operations involving numbers and referents for number, and referents for numbers and quantities” (Understanding, n.d.). However, teachers must assess students’ current understanding for numbers before introducing them to a new set or level of skills. This assessment will ensure students are being introduced to concepts or skills that correlate with or are appropriate for their level of understanding. For example, it is unlikely a student will be successful if he or she is introduced to the concept of place value or problem solving when he or she only holds a foundational understanding for a number’s value. Students who have just been introduced to number sense must first develop an understanding for pre-number concepts. Pre-number concepts include: classifying (ability to sort objects based on attributes); patterns (ability to create, construct, and describe patterns); comparisons (ability to compare the amount of objects); conservation (ability to conserve the value of a number); and group recognition (ability to recognize how many objects are in a small group). Once students have mastered these pre-number concepts, they will be able to think flexibly about numbers and begin to develop strategies for solving complex problems. They will then be introduced to number concepts and skills such as one-to-one counting, counting on, counting backwards, and skip counting. At this level, they will also develop a sense of cardinality, as well as an understanding for terms such as more than, less than, and equal to. Additionally, they will begin to say and write digits correctly as they begin to understand math symbols, such as the addition sign, subtraction sign, and equal sign. Later in their number sense development, they will begin to understand base ten, place value competencies, as well as the ability to manipulate numbers to estimate and solve problems mentally. Through our research as well as our experiences at Longfellow, it has become increasingly clear that there are various stages and levels of understanding number sense. Through exploring these definitions of number sense, as well as several others, we have come to better understand what number sense is as well as what concepts and skills we need to address. We know that with thoughtfully planned lessons that focus on a variety of concepts and skills, our students will develop an appropriate level of number sense which will prepare them for future mathematical learning.

Number sense is important because it allows children to familiarize themselves with numbers and think about numbers in a variety of ways. Students who are struggling in their number sense development have trouble understanding the foundation needed for simple arithmetic, thus these students will struggle immensely when introduced to more complex mathematics in future learning. In 2013, The University of Missouri conducted a study in which researchers analyzed 180 seventh-graders. They found that “those who lagged behind their peers in a test of core math skills[1] needed to function as adults were the same kids who had the least number sense or fluency way back when they started first

[1] Representing a number as different quantities, magnitude, breaking a number into smaller parts, and representing number value on a number line.

grade" (Understanding, n.d.). Research shows the importance of building numeracy early in a child's education to ensure he or she is both prepared and capable of applying critical thinking and problem solving skills in future mathematical learning. However, this understanding does not come by accident; every child can learn number sense, though not every child will unless he or she is intentionally supported through their learning process on an individual basis. It is essential that students receive appropriate instruction and supports based on their current level of understanding in order to further develop or master number sense competencies.

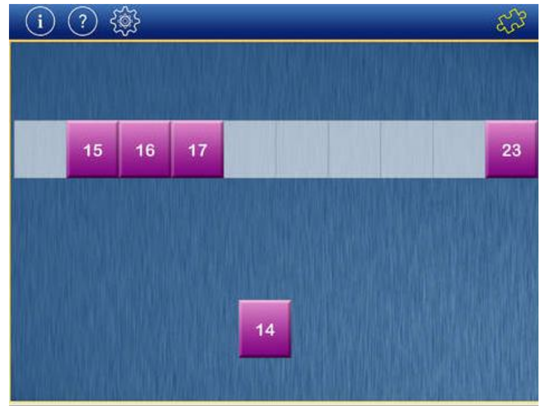
The mastery of number sense concepts varies greatly amongst our Kindergarten students at Longfellow. There are some students who struggle to identify numerals beyond five, while others have already developed more complex number sense competencies such as place value and base ten. Through our work thus far, we have identified three main levels or stages of understanding demonstrated by our students. There are several groups of students within these three "levels". Students who are demonstrating skills within level one have not yet mastered number recognition for numbers one through thirty. Students within level two, though able to recognize numbers, struggle comparing numbers and representing them in ten frames and base ten blocks. Students who are considered to be in level three are developing problem solving skills as well as an understanding of number families. We believe that in order to perform at level two, one must first master all skills and competencies at level one. These foundational skills that are demonstrated at level one, though considered "easier", are essential skills and concepts that students must grasp in order to develop higher thinking skills that will be needed in levels two and three. Because there are multiple groups of students focusing on the same skill, we have had to make accommodations and provide supports to some students based on their individual needs. We have made both accommodations to the content and the process. While focusing on one skill (with students all at the same level), we have modified the lesson's content by adjusting the numbers used in an activity. For example, some students were presented with a different randomized set of numbers when asked to place them in numerical order. In this activity, some students ordered numbers one through ten, while others ordered numbers one through twenty. This modification ensured students were working with numbers that coincided with their current level of understanding. We did not want this activity to go beyond the individual frustration level of these particular students, fearing that they would ultimately reject the activity. Additionally, we have modified the process of our lessons by using a variety of new tools and strategies that we were recently introduced to. These tools and strategies have allowed us to enhance the level of understanding and engagement for students within all three levels. Through this exposure, as well as our observations of student learning, we have become particularly interested in when and how to appropriately implement teaching tools and strategies that will best benefit early childhood numeracy.

Pursing Our Interests through Research and Experience:

Before being accepted into the Number Sense Program, we worked with a group of only four Kindergarten students during our Math Methods course in the fall (September – November). Because at this point, we were only accustomed to mock lesson planning and mock teaching, we were unsure of the tools and strategies that would most appropriately help our students develop their number sense. Throughout our Math

Methods course, we often searched for ideas and activities on the internet, specifically on Pinterest. Looking back on our instructional delivery from the beginning of the year, we realized though many of our activities were "cute" and "fun", they did not provide us with useful means of assessment. Thus, we were unable to collect valid or reliable evidence of student learning. When beginning the Number Sense Program in the winter, we were introduced to a variety of professional teaching tools and strategies that are capable of enhancing the development of number sense that we were unaware even existed. One of our cooperating teachers provided us with "Help Your Students Meet and Exceed the *Common Core Math Standards* for NUMBER SENSE (Grades K-2) – a book she received from a professional teaching conference that contains a variety of lessons and activities that are specifically designed for Kindergarten students working to develop number sense competencies. We were also grateful to receive multiple Classroom Focused Software iPad apps designed by Dr. Hengst and Dr. Egan. Each of these apps was developed to focus on different number sense concepts and skills that have been known to challenge student learning. After being introduced to a variety of both hands-on and technological tools and strategies, we have been able to move away from planning lessons based on generic activities found on Pinterest, as well as outdated activities we remember doing ourselves as young students.

When reflecting on our own experiences as young elementary students, we have come to the unfortunate realization that much of our learning was not interactive or hands-on; rather our teachers often delivered their instruction through repetitive worksheets. We recalled many experiences in which we simply completed worksheets without ever having developed a true understanding of the content or what we were doing. However, we have learned through our multiple methods courses that it is essential that instruction is no longer based on worksheets and rote memorization activities. In our Education Psychology and Measurement (EDUC300) course, we learned of the transfer of learning in light of theorist John Dewey. He found that "in order for transfer to occur, the original learning needs to have been powerful enough… and this of course is largely a function of how the material was taught (as Dewey himself realized, material that was learned because it was being used, in problem-solving for example, is more likely to "be available"; and subsequent research has indeed borne out that active learning is related to successful transfer" (Phillips & Soltis, 2009, 85). Through this gained knowledge, we have come to understand that the way that we were taught as students may not necessarily be the "right" way. We understand that when planning lessons for our students, we must plan activities in which the students can construct their own understanding for the concept or skill through exploration or hands-on learning rather than rote memorization or drill-like instruction. Through these hands-on explorations, students are more likely to be engaged in the activity and thus this "transfer" of learning is more likely to occur. In our methods courses, we have been introduced to new and interactive teaching strategies that are more appropriate for students in this day and age. Students today have access to a larger variety of hands-on materials and technological tools that simply did not exist when we were students. Though we did not have the opportunity to use these learning tools while developing our own number sense, we know it is essential to introduce and expose our students to these tools and strategies early in their numeracy development to better enhance student learning. Because we personally do not share similar learning experiences with our



Kindergarten students, we became increasingly interested in how they work and interact with learning tools such as hands-on manipulatives and iPad apps. Thus, we began to investigate how students developed a particular number sense concept or skill through using hands-on manipulatives compared to how they learned using the iPad apps. We structured the remaining lessons in the Number Sense Program based on this interest. Each week, the students at each level would focus on the same number sense concept or skill for both sessions we met with them. During one session, they would learn the concept or skill through their exploration with hands-on materials. During the other session they would explore the same concept or skill in a similar fashion, however they would learn through the use of a particular iPad app. When planning our lessons, we ensured that not only was the content of both lessons the same, but also the process in which they formed an understanding. For example, one week the focus of our lessons was ordering numbers. On Tuesday we gave an individual set of flashcards to each student asked them to arrange the flashcards in numerical order. On Thursday, students were engaged in a similar process. They were again asked to arrange numbers in numerical order. However, in this lesson the students used the iPad game *Line 'em Up* (As shown in the figure to the right), rather than their flashcards. We found that the *Line 'em Up* iPad app correlated almost identically with our hands-on activity. Because these activities were so similar, we knew we could accurately compare the outcomes of both learning activities. After comparing data of student learning with hands-on manipulatives versus learning with iPad apps we have come to realize that there were situations in which these tools enhanced learning and situations in which they played little to no effect. We also noted experiences in which both of these tools did not engage the learner or contribute to new learning. Thus, we began to question if one tool or strategy was more useful than the other in learning a specific concept or skill. This question led us to further evaluate the learning of our Kindergarten students, as well as research how and when to appropriately use hands-on materials and technology.

Use and Potential Effectiveness of Technology in Schools:

In today's society the use of technology has become increasingly prominent. As technology continues to develop and expand over time, it is beginning to play an essential role in education. Technology has been integrated into schools to guide instruction and reinforce concepts and skills. Research shows that the "goal of placing technology in the classroom is to provide new ways for students to learn. Proper integration of technology will make the technology support these new ways of learning transparently. When students are able to choose and use technology tools to help themselves obtain information, analyze, synthesize, and assimilate it, and then present it in an acceptable manner, then technology integration has taken place" (Ogle et al., 2002, 79). Though most educators agree that technology should be integrated into the classroom, many debate when and how it should be used. “The majority of teachers believe [technology] is effective for motivating students (90%), helping students reinforce or master previously taught content (89%), providing useful information about student learning (66%), and teaching students new content (59%)” (Fishman, Plass, Riconscente, Snider & Tsai, 2015). This controversy and variance in usage has made us question when and how we should use technology in our own instruction. We also began to question how the use of technology should be implemented into the classroom to best enhance student learning.

We have come to understand that students learn best through meaningful, interactive experiences, in which the students develop and demonstrate an understanding of the content on their own, make connections, etc. These meaningful experiences can be obtained from the integration of technology. It is essential that students are given the opportunity to explore technology in playful and creative ways. This will ultimately allow students to be in control of their learning process and make connections to real-world experiences. Additionally, the National Research Council (NRC) stated that technology has the power to “challenge students to solve complicated problems in rule-based virtual worlds [and] have the potential to kick-start the kind of inquiry and project based scientific learning that many education theorists have sought for decades. Such [technology] can help students visualize, explore, and formulate scientific explanations for scientific phenomena that they wouldn’t otherwise be able to observe and manipulate. The [technology] also tends to spark high levels of engagement, encourage repetition and practice and motivate learners with challenges and rapid feedback” (Clemmitt, 2011, 1001). Technology provides students with the opportunity to complete tasks that would be difficult or nearly impossible with the use of physical models, such as altering a set of blocks in terms of how they are assembled or put together. Through our research we have also found ample ways in which technology has the potential to enhance student learning. The use of technology creates a student centered approach to learning in the classroom. This approach allows students to do self-paced learning and take individual ownership of their learning process. Not only is technology useful to students, but it also has the capability to assist teachers. We have found that technology can provide teachers with a better understanding of each individual student’s needs and rate of development. “Computer software that collects detailed information about exactly how a student behaves when taking a test or working math problems can help pinpoint the kind of help the student needs” (Clemmitt, 2011, 1001). Because technology can provide teachers with this information, they are able to recognize the students who are in need of intervention and make appropriate accommodations for these students. Technology has proven to be an effective way to enhance learning outcomes. However, in some learning situations we have found that technology can be relatively inefficient when students have not been properly introduced to these tools.

Although we support the use of technology and feel it is capable of effectively facilitating student learning, we also recognize that these teaching tools can only facilitate learning if and when they are used appropriately. Students must understand how to use these tools before they are expected to develop an understanding for the content in which the tool focuses on. We found that when technology is not used in the “right” way, it does not create positive learning outcomes or provide supports for the learner. For example, when a student has not been properly introduced to a tool, it is unlikely that they will develop a true understanding on their own. Rather, these students may “win” the iPad games using non-related learning strategies, such as blind guessing. Therefore, students are not engaged in the actual learning task nor are they developing a better understanding for the content or intended learning target. When using technology, the biggest challenge students face is that they lose the ability to learn from others as well as the opportunity to construct their own ideas. “When students work on a computer, there is little opportunity for discourse, conjecture, or original ideas. Some software even presents concepts in a fashion as to remove learners from thinking and construction their own understanding” (Van De Walle, 2007, 114). Not only does the incorrect use of technology have the potential to inhibit student learning, but it also has the capability to hinder the teacher’s role and importance in the classroom. In one of our courses, Educational Psychology and Measurement, we learned it is essential as teachers to create a positive learning community and express a personal interest in all students. Therefore, “computers can never replace the human touch in elementary classrooms. Teachers do what technology can’t such as being a live person who cares about you” (Clemmitt, 2011, 1001). When students feel they are cared for by their teacher, they are more likely to feel comfortable and take a greater interest in their learning. Technology limits social interactions with peers and the teacher, which has the potential to negatively influence student learning and teaching. After reflecting on our own experiences, we have found ways in which we have facilitated student learning through the use of technology and ways in which we failed to do so.

Throughout the Number Sense Program, we have made an effort to incorporate the use of technology into at least one lesson per week. We have experimented with several “Classroom Focused Software” apps created by Randy Hengst and Mike Egan such as, *Count Sort, Word Problems, Ten Bead Math, Line ‘em Up, Domino Addition, Balance Math* and *Number ID*. We have found that some of these apps were more useful in certain situations more than in others. For example, we found the *Count Sort* iPad App was particularly helpful for one student who struggled with one to one counting. This app allowed him to move the pieces as he counted. When he practiced one to one counting with manipulatives, he struggled to keep track of which pieces he had already counted, even when he had separated them. However, the *Count Sort* app eliminated this problem. As he counted each piece, it changed to a different color, which helped him recognize which pieces he had counted and which he had not. With the use of this app, we were able to address the needs of this particular student and assist him in developing this skill. Other apps, such *Line ‘em Up* and *Domino Add*, increased the students’ level of engagement and motivation to learn. While students explored these apps, they were in control of their learning because they were allowed to independently complete these tasks and construct their own understanding. Not only are both of these apps engaging and easy for students to use, but they are also useful because they can be differentiated, which allowed all students to be successful in developing an adequate understanding of the concepts. However, we found that some apps did not engage students while others required additional instruction and attention throughout the lesson. Thus, students were unable to independently develop an understanding for the concepts. For example, when students were introduced to the *Balance Math* app, they were distracted and unfocused. During this lesson, the students continuously attempted to balance the “beam” by placing the exact same number or “weight” on each side of the beam. For example, if a 4 was placed on one side of the beam, the students would place a 4 on the opposite side, rather than two addends that have a sum of 4. This app required the students to think of two addends and multiple fact families on their own. This is not something our students were accustomed to. Our students have always been provided with manipulatives or other teaching tools to assist them in their problem solving strategies. We do not feel that the students held an adequate understanding for the concepts needed to engage themselves in using this app appropriately. Because they lacked this understanding, students were balancing the numbers by guessing or recognizing patterns in the colors. Thus, we were unable to collect valid data from this lesson or use the iPad app as a useful assessment tool because the students were getting the correct answer without demonstrating a true understanding for the concepts. Additionally, we noticed that when using some apps, such as *Ten Bead Math*, the students were unable to independently explore this app. The students were unsure how to use this app to help them solve the addition equations. When they attempted to explore this app, the chips were too small for them to use appropriately. Though this is not necessarily a flaw in the app, it requires the teacher to spend more time teaching how to use the app and more time for students to practice using the app before they are expected to develop an understanding through its use. Because we did not provide our students with an adequate amount of time to freely explore the app, they were in need of direct instruction and guidance throughout the whole lesson, which eliminated the possibility of the students constructing a more meaningful understanding on their own. Because the students were not given much time to practice using the app and moving the smaller pieces, they were unsure of the app's purpose and how to correctly use its features to represent the addition equation. Thus, the students resorted to traditional and familiar problem solving methods, such as using their fingers to add when the sum was less than ten. We found the students struggled immensely when the sum was greater than ten because they did not have enough fingers to represent this equation. We hoped that the students would utilize the features on the *Ten Bead App* during this lesson, but many were unsuccessful and seemed uncomfortable using this app. Through our research and own experiences in the Number Sense Program, we have recognized and developed a better understanding for how to facilitate student learning and ensure technology is being appropriately used.

Use and Potential Effectiveness of Manipulatives in Schools:

Before technology, specifically mathematical iPad apps, had emerged into the classroom, the most traditional method for problem solving was through the use of manipulatives. Manipulatives are known as physical objects used to engage students in their learning process or simply objects to think with. Similarly to what researchers say about iPad apps, Swan and Marshall state that "a mathematics manipulative material is an object that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered" (McIntosh, 2012). These hands-on interactions have the potential to help students solve problems independently because they assist the students in representing the problem and constructing their own thinking. However, this depends on the materials available, the ability of the student to use the materials, and the appropriateness of the materials for representing the problem. "Manipulatives can be used in teaching a wide variety of topics in mathematics, including the objectives from the five National Council of Teachers of Mathematics (NCTM) standards: problem solving, communicating, reasoning, connections, and estimation. The materials should foster children's concepts of numbers and operations, patterns, geometry, measurement, data analysis, problem solving, reasoning, connections, and representations" (Boggan, Harper & Whitmire, n.d.). Because manipulatives can be used in teaching various mathematical concepts and skills, it is essential that teachers are able to provide a wide variety of manipulatives to their students. Manipulatives can range from objects such as counting chips, measuring instruments, base-ten blocks, balances, and countless others. We have found that there are many different types of manipulatives that can be used to address numerous mathematical concepts and skills; however, we have come to understand that what is most important is how students interact with the manipulatives and the kinds of ideas they are exploring during these interactions. As teachers observe their students' interactions with manipulatives, just as they do with iPad apps, they must recognize their emerging mathematical ideas and encourage their students to reflect on their play and construct meaning through their experiences. Without these student-teacher interactions, teachers will struggle to accurately assess the students' current level of understanding, developed through the use of manipulatives. Manipulatives have the potential to be beneficial in developing mathematical concepts and skills; however there is evidence that proves the use of manipulatives can sometimes be relatively inefficient to student learning when used to address the wrong concept or skill or when students are unsure of its use.

After working with our four Kindergarten students in the fall (September – November), and reflecting on the teaching strategies we used, we felt confident implementing manipulatives into our lessons during the Number Sense Program. We knew these teaching tools would increase students' level of engagement, motivation, and understanding. We also knew they would serve as effective objects to think with that would allow students to represent a problem and visually construct an understanding. Through our research, we discovered many other ways in which manipulatives have the potential to greatly benefit student learning. We have found that "according to many researchers, the use of auxiliary means, such as manipulatives and representations, are crucial elements of high quality mathematics education at all levels. This is because manipulatives and representations are particularly useful in helping children move from the concrete to the abstract level" (Skoumpourdi, 2010, 150). Students are able to make this move because they are physically constructing an understanding that can be visually seen, as well as developing a deeper understanding through communication with their peers. We have come to understand that all students are able to move from a concrete to abstract level, or benefit from the use of manipulatives, despite their unique intelligence level. In our methods course, Methods of Inclusion, we learned that Howard Gardner identified several distinct intelligences including visual-spatial, bodily-kinesthetic, interpersonal, intrapersonal, musical, logical-mathematical, linguistic, naturalistic, and existential which can all be addressed through the use of both technology and manipulatives. Through our research and experience, we feel that because there is a multitude of ways to implement manipulatives, they have the potential to enhance the learning process for students of all intelligences. For example, when using counting chips to solve a mathematical equation (logical-mathematical), students will be physically constructing an understanding (bodily-kinesthetic), which they will then be able to visually see (visual-spatial). Through this exploration, students will be independently developing their own understanding (intrapersonal), while also collaborating with peers and sharing their understanding and process of development (interpersonal). Manipulatives also have the capabilities to address linguistic and musical learners in other learning situations, such as through interactive literature reading or word play (linguistic) and by associating music and sound to mathematical thinking (musical). Both manipulatives and technology are capable of appealing to multiple intelligences. However, we have come to understand that the experiences students have when moving chips on a screen as opposed to moving tangible chips in front of them differ immensely. Because manipulatives are concrete teaching tools, they allow students to construct their own understanding in a way that is meaningful to them. Manipulatives can be used in a variety of ways, however "math is best understood and appreciated by children and teachers alike if it follows the five C formula: collaborative, concrete, comprehensive, connecting, and cavorting (Murray, 2001, 28). The five C formula focuses on making math action based where students work together to solve math challenges (collaborative); making math tangible where children learn through their senses (concrete); making math ubiquitous where children know how and why they are learning which reduces levels of anxiety (comprehensive); making math an experience where children are involved in their learning and able to make connections to their lives inside and outside the classroom; and lastly making math fun where children are engaged in play and do not realize they are learning, which makes their learning more memorable (cavorting) (Murray, 2001, 28). When manipulatives are used this way, they are capable of assisting students in their learning process, despite their level of intelligence. Because we felt confident about using manipulatives while we learned as elementary students ourselves, as well as through our teaching at Longfellow, we were surprised at the ways in which the use of manipulatives can be relatively inefficient to student learning if they are not integrated in an appropriate manner.

After reflecting on our own experiences in elementary school, as well as specific experiences in the Number Sense program in which students developed and demonstrated a greater understanding for mathematical concepts and skills, we strongly felt that manipulatives were the best strategy to effectively facilitate student learning. However, through our research and further evaluation of our lesson outcomes, we are aware there are situations in which student learning through the incorrect use of manipulatives may be less than optimal. Incorporating manipulatives into the classroom takes time, can be expensive, and may be difficult to use in a large classroom setting, or a classroom with behavioral and management issues. Manipulatives provide students with the opportunity to freely and independently explore mathematical concepts and skills. However, we have come to understand that children are likely to be unaware of their mathematical development, and fail to construct an understanding through the use of manipulatives. Without peer and teacher interactions, students may be simply "playing" with manipulatives. Manipulatives do not automatically present the user with the correct answer through their use, and therefore it is clear that the value of manipulatives depends on how they are used to solve problems (Kamii, Lewis & Kirkland, 2001, 21-31). Though manipulatives do not automatically present users with the correct answer, this is not a reason to not implement them in your classroom. Students will find value in their learning and development of their understanding when given the opportunity to use the manipulatives to construct an understanding on their own. However, manipulatives will not be useful to students if they are unsure of how to appropriately use them to develop an understanding of the content. Thus, if students do not have an adequate understanding for their use or purpose, they will only develop further misconceptions. For example, students often develop a misunderstanding for representing place value through the use of manipulatives. When students see the symbolic number "52" they see this number represented as "fifty two ones". Adults, however, can think about this number as "fifty two ones" and "five tens and two ones" simultaneously. Young students who are being newly introduced to place value have not yet developed the ability to think simultaneously about numbers as adults do. For example, students were asked "to put ten beads into each cup as shown in the figure [to the right], and when asked to count all the beads by tens, they often say "10, 20, 30" as they count the cupfuls and go on to count the loose ones by saying "40, 50, 60, 70." Adults and older children know when to shift to *ones* in this situation because they are thinking about the *ones* while counting by *tens*... Therefore, it is not possible for children to acquire a system of *tens* empirically from base-ten blocks, Unifix Cubes, or bundles of straws or toothpicks". Mathematics does not exist in manipulatives, but rather it develops by how children interact with and construct an understanding through the use of these teaching tools (Kamii, Lewis & Kirkland, 2001, 21-31). It is clear that when using manipulatives, students may develop misconceptions or fail to develop an understanding entirely. Thus, teachers must ensure students understand the manipulatives purpose as well as how to properly use the manipulatives to solve mathematical problems. After reflecting upon all research gathered, we compared the information that both supported and disapproved of the use of the manipulatives to our own teaching experiences in the Number Sense Program. We were able to recall specific situations in which we facilitated student learning by using manipulatives appropriately as well as situations in which our students failed to meet our learning goals because we had inappropriately implemented manipulatives into our lesson.

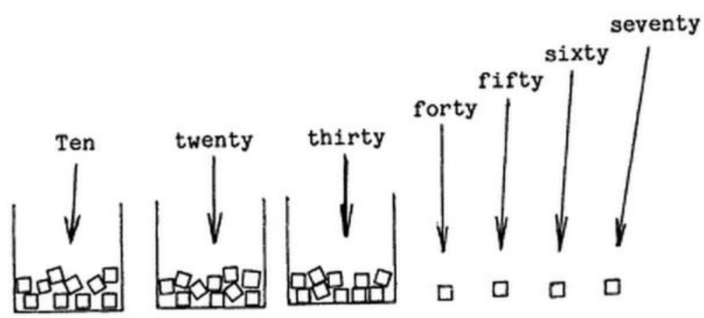


Fig. 8.
The way many first graders count three cupfuls and four loose beads.

Similarly to how we implemented technology into our lessons, we also incorporated various manipulatives into our lessons once a week. Students explored several manipulatives including counting chips, balance beams, base ten blocks, connecting cubes, flashcards, dominos, a Rekenrek, and other tools that allowed for hands-on interactions (writing numbers in shaving cream, crafting "turkey fingers", etc.) We found that some of these teaching tools were more successful in some cases than others. For example, when working with students on skills such as number recognition and writing numbers, we provided each student with a small pile of shaving cream. They were to use the shaving cream to write the number we had orally stated to them. These students in the past had typically used paper and pencil to practice hand writing numbers, but by breaking their routine we felt that this activity increased their level of understanding and retention of the information, which we observed in both previous and following lessons. Furthermore, we noted that when students used counting chips to practice basic addition skills they were more successful than when they had solved addition problems without the assistance of manipulatives. We feel that students more accurately and efficiently solved these equations because they were able to use the manipulatives to first represent the equation and then solve it. For example, when given the equation “5+8=?” students first counted out and placed five chips (with the red side facing up) in one pile and eight chips (with the white side facing up) in another pile. The students recognized these two separate piles as addends and knew they must add them together to determine the total number of chips. It was clear that counting all the chips allowed students to create a visual representation and assist them in their problem solving. In weeks following, we provided students with a different manipulative that would be used to again solve addition equations. We provided each student with a “homemade” single stand Rekenrek (Rekenrek, 2010), created using a pipe cleaner strung with ten white beads on one side and ten red beads on the opposite side. Though we asked the students to solve the same addition equations they had solved using the counting chips, there was a recognizable difference in their level of understanding of problem solving approach and capabilities. Many students who are just beginning to demonstrate an understanding for problem solving “rejected” this manipulative, just as they did with the *Ten Bead Math* app, and instead made use of a more traditional and familiar approach to problem solving. We allowed the students to independently solve the addition problems using the problem solving strategy that they preferred. However, we noticed that when the sum was greater than ten, the students who relied on the use of their fingers struggled immensely in solving these problems because they were unable to represent the equation on their fingers. Even when encouraged or prompted to solve the equations using the Rekenrek, the students again seemed uncomfortable with this teaching tool because we did not allow them enough time to explore and develop an understanding for its purpose or use. We also identified other lessons in which students were not interested in exploring the manipulatives presented, but rather they used more familiar and comfortable methods that would assist them in discovering the answers in the quickest way possible. We found that it was difficult for students to “unlearn” a traditional learning method to explore new ways of problem solving through the use of manipulatives. Through our work in the Number Sense Program and our research on manipulatives, we have become familiar with the ways in which these teaching tools can enhance student learning when used appropriately.

Only Right When Used Right:

Through our work in the Number Sense Program, we have observed a variety of ways in which children can construct their understanding of number sense. We have come to understand that all children, even within the same “level”, solve problems in a way that is preferred by them. Because each student has a learning style that is unique to them, it is essential for teachers to recognize and except multiple ways in which children can solve the same problem, understand that not all students will “get it” at the same time, and that students may prefer to use different materials. “If we are to make math experimental, we must provide children with tactile tools with which they can learn, opportunities to interact with each other and the teacher, and diverse methods of arriving at the correct answer" (Murray, 2001, 28). As referenced in *Perspectives of Learning,* Dewey claims it is the job of the teacher to create a stimulating environment in which all children can learn in the way that is most beneficial to their unique learning needs (Phillips & Soltis, 2009). We have found that some students prefer using manipulatives while others found the use of technology to be more helpful. Because no two students learn exactly the same way, teachers must create a balance between implementing manipulatives and technology into their classrooms. When planning lessons, teachers may ask themselves which teaching tool would be most useful and appropriate for the concept or skills being taught. We know that the use of manipulatives and technology can facilitate student learning, however we have come to understand that these teaching tools can only be beneficial to the learner if they are integrated appropriately into instruction. Thus, through our research and own experiences, it is clear that these teaching tools are "only right when used right". Teachers can make the "right" use of these tools by providing enough time for students to experiment and explore new tools. Without this introduction, it is difficult to determine whether the student is struggling with the mathematical concept or how to properly use the tool. "Children need time to explore the functionality of technology before they can be expected to use these tools to communicate. Just as we encourage children to use crayons and paper well before we expect them to write their names, it seems reasonable to provide access to technology tools for exploration and experimentation" (Technology, 2012, 3). If the student does not know how to properly use the tool, it is likely the student will not be able to develop an understanding for the mathematical understanding at all. In order for students to use the tool in the "right" way, it is essential that teachers allow for social collaboration and monitor their students' use. They must ask students to reflect on their experimentation with the teaching tool to ensure that it is being used the "right" way and that they are developing an adequate understanding. The students' responses and teacher's feedback will then give teachers a better insight as they reflect at the end of the lesson on whether or not the teaching tool was used "right". These student-teacher interactions will help the teacher determine what gains were resulted from the use of the teaching tool, which may not have been otherwise achieved with the use of another. In order for these tools to be used "right" throughout the entire lesson, it is essential that both the teacher and the students are knowledgeable on how to properly use them.

Without proper training on how to utilize teaching tools, such as technology and manipulatives, it is likely that both the teachers and the students will fail to use them in the "right" way. Teachers may struggle to implement these tools in a way that would most benefit student learning. Without proper training, teachers may find it difficult to choose the most appropriate tool for the concept being addressed. In other words, they may fail to see what one tool can do, which another tool cannot. Thus, teachers must be properly trained and educated on how to use the variety of teaching tools available to them. The National Association for the Education of Young Children states that educators must "have the knowledge and experience to think critically about the selection, analysis, use, and evaluation of technology and media for young children in order to evaluate their impact on learning and development" (Technology, 2012, 3). Many teachers lack the knowledge necessary to implement technology the "right" way into their classroom because they believe that in order to obtain this valuable knowledge, they must attend technology courses, complete coursework, prepare a portfolio, or attend a professional teaching conference. However, there are many resources readily available for teachers to develop this understanding; such resources can be found on the internet, in teaching manuals, or on app reviews and comments. Teachers can however develop the knowledge needed to implement technology by first fully understanding the use of just one app. Similarly, teachers must have an adequate understanding for how to implement manipulatives in the “right” way. Educators may not realize that they need to develop an understanding of the appropriate ways in which manipulatives can facilitate student learning. Unless teachers have been trained or educated, “they are likely to make only token use of them, which may be detrimental to learning” (McIntosh, 2012). Rather, teachers must create enriching experiences with manipulatives and apps for their students in order to eliminate any previously learned misconceptions and build their confidence for further manipulative use. Not only do teachers need to be properly trained and educated, it is essential that students are as well. If teachers give students teaching tools, such as technology and manipulatives without a proper introduction or time for exploration, it is likely that these students will be “playing” with these tools without developing an understanding by the end of the lesson. Because children may be relatively unfamiliar with various technological tools, they may lack an understanding of how to appropriately utilize them. Therefore it is vital that students are allotted time to practice using these tools before they are expected to use these tools independently – for example, dragging chips on the screen. Without developing the understanding of how to use these tools prior to the lesson, students may struggle immensely throughout the lesson, and as a result fail to reach the end learning target. Students must also have an adequate understanding of the “right” ways in which manipulatives should be used. Because one specific manipulative can be used in different ways to address different concepts, it is essential that students understand what the manipulative is representing or symbolizing. Educators must realize that “repeated exposure to manipulatives is necessary for students to understand how to use manipulatives and feel comfortable with them” (McIntosh, 2012). Without this exposure and confidence in using manipulatives, it is likely that students will fail to develop an understanding for the content that was intended to be taught through the use of the given manipulative. Thus, it has become clear that both teachers and students need to be trained and informed of how to properly utilize technology and manipulatives in the “right” way.

Through our research and experiences in the Number Sense Program, we believe technology should support instruction rather than replace it. Technology allows for better access to instruction, thus it is essential it is infused into the classroom. Although technology is becoming more prominent in schools, teachers must first understand that their students can only benefit if it is used in the “right” way. After collaborating with Hengst and Egan, we have come to understand that none of the Classroom Focused Software apps were created with the intent of introducing a new skill or concept. Rather, these apps were designed for additional practice. It is necessary that teachers understand the “right” way to implement these tools into their classroom. We feel that the “right” way to implement technology is to do so in a way that supports instruction rather than replaces it. For example, the A-Games Project conducted multiple studies through the

University of Michigan, in which a 4th grade teacher was interviewed. She stated that she uses "the games primarily to reinforce skills and provide high-engagement practice, particularly when students have been working on the same skill for a long time and are growing restless. [She] also uses digital games as a reward" (Fishman, Plass, Ricon, Scent, Snider & Tsai, 2015). Hengst and Egan explained that they designed their Classroom Focused Software apps with the intent to reinforce already learned concepts and skills. After discussing the "right" way to implement iPad apps, we developed a deeper understanding for how children make mental connections between the content to be learned and the teaching tools being used in the lesson. They further explained Piaget's stages of development and we came to realize the importance of students developing an understanding with concrete objects in front of them. As previously stated, moving chips is a more concrete process than dragging chips on a screen, thus it is difficult for students to make these mental connections if they have not explored the manipulatives first. Hengst and Egan admitted that "apps are sometimes just too big of a jump". If teachers desire to implement technology into their lessons, students should first be given an opportunity to explore the content through the use of manipulatives. Through our research and own experience, we have come to understand that the use of technology can be used in the "right" way if it supports instruction and further helps students construct an understanding.

Incorporating manipulatives into a mathematical lesson has the potential to greatly benefit student learning, however, this is only possible when they are used in the "right" way. Children must first understand the mathematical concept being taught. Without developing even a basic understanding for the concept, they will simply be moving the manipulatives around. Though it is essential for students to have an understanding of the concept, they must construct this understanding on their own. If students simply observe a demonstration of the use of manipulatives, they will fail to develop an adequate understanding because they are not engaged in the material or actively manipulating the tool on their own. Research shows that when children are using manipulatives in the "right" way, they are capable of improving their long-term and short-term retention of math (Boggan, Harper & Whitmire, n.d.) When students are involved in hands on learning and actively construct an understanding on their own, their understanding holds more meaning which is more readily retained and recalled. Additionally, research suggests that children at a young age may receive more benefits through the use of manipulatives than upper elementary students. According to developmental theorists, "children in early childhood (age 7 and younger) should benefit from exploring mathematical concepts with manipulatives… The reason for this expectation is that younger children are assumed to have a greater dependency on physically interacting with their environment to construct meaning" (Carbonneau, Marley & Selig, 2013, 385). Hengst and Egan designed the Classroom Focused Software apps with this idea in mind. They intended for the apps to be used by the Kindergarteners after the use of manipulatives. Older students, however, who have developed the ability to reason abstractly are more capable of performing mathematical tasks that consist of symbolic representation. On the other hand, young children are "predicted to experience more difficulty when provided instruction that solely consists of symbolic representation; therefore, the assumed cognitive benefits of manipulating concrete objects to represent mathematical concepts should be greater for younger children who are still developing proficiency with higher level representations" (Carbonneau, Marley & Selig, 2013, 385). Manipulatives are essential tools that assist all students in constructing their own understanding. However, research indicates that manipulative use in grade level decreases as age and developmental processes increase. Therefore, it is essential that teachers are made aware of this decline to ensure they are implementing manipulatives in the "right" way.

When reflecting on our lessons with our kindergarten students, we have recognized the ways in which we have successfully facilitated student learning through the use of manipulatives and iPad apps. For example, we feel strongly that we implemented the iPad app *Line 'em Up* in the "right" way because students were able to construct an understanding independently. We used this app as a tool to assist students in comparing and ordering numbers. This app presented students with a symbolic number and required them to place the numbers in numerical order on a number line. If students incorrectly sequenced a number, the app would reject their answer, and force them to make additional attempts until it was correct. Because of this feature and helpful feedback, students were able to develop a deep understanding for this concept independently. In the following lesson, students were again asked to compare and sequence numbers using their flashcards. We found that more students struggled with independently completing this activity because they were unaware of their mistakes when sequencing the numbers in front of them. We also recognized ways in which we enhanced student learning by implementing manipualtives in the "right" way. As students continued working on comparing numbers, they were given connecting cubes to represent the quantities of two numbers. The students were asked to make two "towers" and identify which sum was greater. Most students did not struggle with this task because they knew the bigger or taller "tower" was more. However, some students were faced with a challenge when asked to state how many more cubes were in one tower compared to the other. Because we observed a greater struggle with this task, we thought of ways in which we could use the manipualtives to assist the students. We encouraged them to lay one "tower" on top of the other, matching up quantities that were the same. This allowed students to more easily recognize the "left overs" as the difference in the two numbers. Although there were many ways in which we implemented teaching tools in the "right" way, there was several times in which the use of these tools "failed" to facilitate learning. However, through our research and extensive collaboration with Egan and Hengst, we have come to understand that this "failure" was not due to a fault in the tools, but rather a fault in the ways in which we implemented them. Despite our conscious efforts, however, there were lessons in which we failed as teachers to appropriately implement these teaching tools in a way that would best benefit the learner. For example, we had the students explore the *Count Sort App* set to the Ten Frame setting. This app challenged students to identify the teen numbers being represented in a ten frame. The students were only presented with two options – the correct answer and its' reversal. For example, if the number seventeen was represented in the ten frames, the students had the option of selecting either "17" or "71". Thus, we found this app to be incredibly easy for the students. We met with Hengst and Egan to discuss this "fault" and discovered that we failed to realize there were additional features that allowed the user to change the options presented. Instead of providing the students with only two choices, we could have provided them with three or four. Though we failed to implement this app in the "right" way, we were able to learn from this experience as we now know that we must explore all features of the app before utilizing them in our lessons. Additionally, we have identified other ways in which we failed as teachers to implement teaching tools in the "right" way. During a particularly unsuccessful lesson, we recognized the confusion and frustration in our students. The students were given a balance beam and were asked to place a weight on a number on one side of the beam. The students were then asked to "balance" the beam by placing two weights on the opposite side. However, many students failed to recognize that one weight dangling represented the quantity of a number, rather than one object as they observed. For example, they were unable to understand that when one weight was placed on the number three it represented the quantity of three or that when two weights were placed on the number four, it represented the quantity of eight rather than two. Many of the students were unable identify the quantities represented by weight because their focus was simply on the number of objects being used. To help with this confusion, we provided the students with chips to split up into piles to represent the quantities. For example, if one weight was placed on the number six, we provided the student with six counting chips and asked them to separate them in piles. We hoped this would help the students discover two possible addends that could be used to balance the beam. However, even with this modification, the students continued to struggle. After reflecting on our lesson and our observations, we believe that the problem was not in how the students used the tools, but rather in how we implemented it. We chose to use this tool to address addition and fact families. However, through our research we have come to realize this was not the "right" way to implement this tool. We found that balances are appropriate for teaching the measurement of weight, not addition. No one learns addition from a balance because addition is a mental operation, where as a balance "is a physical phenomenon and is not the same thing as the logicomathematical relationship of equality" (Kamii, Lewis & Kirkland, 2001, 21-31). Because we chose to implement the balance beam for the wrong concept and skills, the students were unable to develop an adequate understanding through the use of this teaching tool. After reflecting on our lessons throughout the Number Sense Program and comparing our outcomes to our research findings, we were able to better identify lessons in which we used teaching tools in the "right" way as well as lessons in which we did not.

Focus Students:

After working with our students for several weeks, we recognized a noticeable range of understanding amongst our students. We noticed that some students had already developed problem solving capabilities while others could not recognize numbers one through ten. Thus, some of our initial activities proved to be too easy for some students while others struggled immensely. Because of this range of abilities, we separated groups into three "levels". These levels were based on their current level of understanding for numeracy. As teachers, we know that we must differentiate our lessons and cater to our students' individual needs. However, we knew that because these students did not have the same understanding for numeracy, we had to plan activities and lessons that addressed the concepts and skills that were most appropriate for their current level of understanding. For example, it would be inappropriate for a student at “level one” to do a problem solving activity when he or she struggles to identify the numbers or represent the quantities in the equation. Thus, we came to realize that we must help these students develop their number sense by building on their current level of understanding. We first determined the concepts and skills that would be addressed at each level. Once we determined these concepts and skills, we sought out a variety of strategies and teaching tools that would enhance our teaching in a way that would most benefit each individual learner. Because no student learns in the same way, we have come to realize that as teachers, it is essential our lessons appeal to multiple learning styles. Thus, we structured our lessons in a way that would allow students to explore a concept or skill through the use of both technology and manipulatives. Students were to use these teaching tools to develop a greater understanding and further develop their number sense. Once a student has mastered the concepts or skills within their level, he or she could move up a “level” where they will face additional challenges. Level one students focused on identifying numbers, while level two students focused on understanding number value and level three students focused on applying their number sense. We selected one student within these three levels to serve as our focus student. We chose these students so we would be able to more closely observe and monitor the progress of a specific student at each level of number sense development. Each week, we videotaped the students during the lesson. We later reviewed and analyzed these videos to ensure we did not miss any cues or behaviors while teaching. We also periodically administered the ESGI Assessment to these students to collect data and document their growth. Though we were planning and delivering multiple lessons a day, we knew we were planning the most appropriate activities that would benefit our focus students as well as the students within their group.

Students that were placed in level one began by focusing on identify numbers. Our goal for them was to develop number awareness. We hoped that by the end of the program they could identify all numbers one through thirty, successfully count to one hundred and count on from a given point as well as develop one-to-one correspondence. We knew that having an understanding for these basic math and number concepts was essential before moving onto more advanced math concepts (level two and level three). Students need to build a foundation for these basic skills before they are exposed to more complex material. Students first need to understand what the numbers look like. This visual discrimination is key to developing other number skills such as counting. When first learning to count, a child relies on rote memorization for numbers one through ten. Most students early in their numeracy have yet to develop an understanding for cardinality, or that the last counting word tells how many. For example, a child can count seven objects but when asked how many objects he or she had counted they are likely to re-count the objects, rather than simply stating seven. Once students are able to successfully count forward starting from one, they should be challenged to start counting forward from other given points. A student should be able to start from six and follow by saying “seven, eight, nine” without having to count forward from one. Counting on allows students to continue counting objects added to a previously counted group without recounting the entire group. For example, a student who has mastered counting on can count three objects and when given two more he or she can continue counting (four, five) rather than re-counting previously counted objects or starting from one to count up to five. As students develop rote-counting abilities, they should also be challenged to apply their understanding or ability to count to counting a pile of objects. Students must develop one-to-one correspondence, or the ability to label one number to each object. A student should touch each object or separate the objects counted from the ones that have not yet been counted to ensure he or she does not count one object twice or fail to count it at all. These foundational skills – number recognition, rote counting, counting on, and one-to-one correspondence – are essential to develop early on in childhood numeracy. Thus, these skills became the focus of our students in level one.

When we first met focus student one, we quickly recognized this student’s lack of understanding. It was clear that this student was much further behind in his numeracy development according to observations made by his classroom teacher. He had not yet developed an understanding for the concepts or skills needed at the most basic level of number sense. As we assessed his current level of understanding it was obvious that this student could only recognize numbers one through five and count to ten. He could not count on from a given point or demonstrate one-to-one correspondence. Because this student lacked an adequate understanding from the beginning, it was essential that we planned lessons based on his current level of understanding as well as provided him with the appropriate supports. We helped this student develop his number sense by incorporating both technology and manipualtives into the lessons. For example, when this student focused on number recognition, he explored iPad apps such as *Number ID* and *Line ‘em Up.* He also used other tangible manipulatives such as flashcards, “goo” bags and shaving cream. Although we provided the student with multiple teaching tools, he did not seem to prefer one to the other. Student one never demonstrated a deeper understanding or a gestalt moment through the use of an iPad rather than manipulatives, or vice versa. Though he did not prefer one tool to the other, he was able to use these tools to deepen his understanding of number sense. After working with the student for over 20 weeks and analyzing his assessment data (see below) it is clear that this student has strengthened his understanding for numeracy. Student one can now recognize most numbers one through thirty. This student still confuses numbers such as twelve and fifteen, as well other teen numbers. However, the data collected from his assessments proves that his ability to recognize numbers has greatly improved throughout the course of the year. Through developing the ability to recognize numbers, it is apparent that this student has also developed an understanding for the number’s quantity, which has helped him successfully be able to order and compare numbers. Though this student has improved immensely, he still is lagging behind his peers. We hope that with this newly developed understanding for these foundational number sense concepts and skills, this student will be more successful in developing an understanding for more advanced mathematical concepts.

Student 1: ESGI Assessment Data

	FALL	WINTER	SPRING
Number Recognition	2 out of 32 correct	11 out of 32 correct	20 out of 32 correct
Rote Counting	1 out of 10 correct	1 out of 10 correct	3 out of 10 correct
Comparing Numbers	4 out of 8 correct	3 out of 8 correct	8 out of 8 correct
Recognizing Shapes	9 out of 18 correct	7 out of 18 correct	14 out of 18 correct
Counting Objects	N/A	12 out of 31 correct	14 out of 31 correct
Writing Numbers	1 out of 30 correct	8 out of 30 correct	21 out of 30 correct

Student 1: ESGI Assessment - Fall

RI- Numbers Q1
(2 out of 32)

Correct	Incorrect
4, 5	0, 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31

RI- Counting Q1
(1 out of 10)

Correct	Incorrect
10	20, 30, 40, 50, 60, 70, 80, 90, 100

RI- Compare Numbers Q1
(4 out of 8)

Correct	Incorrect
1 3, 3 2, 4 5, 5 2	10 8, 6 9, 7 3, 8 10

Opt. K Qtr. 1 Shapes
(9 out of 18)

Correct	Incorrect
Circle, Cone, Diamond, Heart, Hexagon, Moon/Crescent, Star, Trapezoid, Triangle	Cube, Cylinder, Octagon, Oval, Pentagon, Rectangle, Semi-Circle, Sphere, Square

Student 1: ESGI Assessment - Winter

RI- Numbers Q2
(11 out of 32)

Correct	Incorrect
0, 1, 2, 3, 4, 5, 6, 7, 8, 22, 27	9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 28, 29, 30, 31

RI- Counting Q2
(1 out of 10)

Correct	Incorrect
10	20, 30, 40, 50, 60, 70, 80, 90, 100

RI- Compare Numbers Q2
(3 out of 8)

Correct	Incorrect
3 2, 5 2, 7 3	1 3, 10 8, 4 5, 6 9, 8 10

Quarter 2 Shapes
(7 out of 18)

Correct	Incorrect
Circle, Cone, Heart, Moon/Crescent, Oval, Star, Triangle	Cube, Cylinder, Diamond/Rhombus, Hexagon, Octagon, Pentagon, Rectangle, Semi-Circle, Sphere, Square, Trapezoid

Student 1: ESGI Assessment - Spring

RI- Numbers Q3
(20 out of 32)

Correct	Incorrect
0, 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 13, 14, 16, 17, 18, 20, 22, 25, 26	7, 12, 15, 19, 21, 23, 24, 27, 28, 29, 30, 31

RI- Counting Q3
(3 out of 10)

Correct	Incorrect
10, 20, 30	40, 50, 60, 70, 80, 90, 100

RI- Compare Numbers Q3
(8 out of 8)

Correct	Incorrect
10 8, 3 2, 4 5, 5 3, 5 2, 6 9, 7 3, 8 10	

Quarter 3 Shapes
(14 out of 18)

Correct	Incorrect
circle, cone, crescent/moon, cube, heart, hexagon, octagon, oval, rhombus/diamond, semi-circle, square, star, trapezoid, triangle	cylinder, pentagon, rectangle, sphere

Once students have mastered all concepts and skills that are addressed under level one, they will be ready to move onto more complex material that is associated with level two. Our level two students focused on understanding a number's value. Our goal for these students was to develop an understanding or the ability to represent numbers using ten frames or base ten blocks, understand place value, and compare numbers. Students who are in the midst of their number sense development should be challenged to apply their understanding for numbers and quantity to how they should be represented. Students must understand that numbers are composed of tens and ones, which will ultimately prepare them when working to determine the place value of larger numbers. Students can develop an understanding for this concept through the use of ten frames. Students can arrange counters in different ways on the ten-frame to visually see what the number is composed of. A ten-frame can also be used to help students compare numbers. They can use their arrangements to easily see that six is more than two and four is less than seven. Students can also explore the concept of place value through the use of base ten blocks. These 3-dimensional blocks can be manipulated to represent and express the value of numbers. However, when using these blocks, students must understand the quantities that each block represents. For example, if asked to represent the number 25, the student must understand that the two longs represent ten units each, while the five units represent five ones. Students must learn how a number is composed as well as its relationship to the number system in order to represent the numbers' place value. Once a student has developed an understanding for place value, he or she will be able to interpret and compare these numbers, gaining a broader understanding for the number system as a whole.

We considered student two to have an "average" level of understanding based on his assessment data and conversations we have had with his classroom teacher. After working with this student for several weeks, it was clear that this student had mastered level one concepts and skills such as number recognition, rote counting, and ordering numbers. Thus, we determined that this student had developed the foundational numeracy skills needed to explore more advanced concepts such as number value and place value, as well as representing numbers in multiple ways. Student two also explored these concepts through the use of both manipulatives and technology. When focusing on a skill such as place value, student two used the iPad app *Math Tools* to explore base ten. In a separate lesson focusing on the same skill, he also used tangible base ten blocks. Through conducting research we have learned that students need to be exposed to one teaching tool multiple times. We wanted to test this idea on one of our focus students. Typically, we had our students explore a concept using an iPad app on one day and manipulatives on the other day in a week. We allotted student two the opportunity, however, to use the same tool (both the manipulative and the app) twice in a week before exposing them to the other tool. Although he was only given one extra day to explore the tool, we are confident that these additional exposures helped him develop a deeper understanding for the concept compared to other lessons where he was given less time to explore. Throughout the course of the Number Sense Program, student two was provided with a variety of teaching tools to assist him in developing an understanding for number value. For example, student two focused on representing a number's value by using a new iPad app *Make Another B*, which we helped Hengst and Egan create. He did a similar activity using connecting cubes. We feel both the iPad App and manipulatives helped student two develop an understanding for the skills needed to "perform" at level three. After reflecting on the Number Sense program, we feel that student two has grown immensely. He is now able to represent numbers and their values in multiple ways. Not only has his number sense capabilities been strengthened but we also feel that this student has developed a more positive attitude or outlook on math. He seems much more confident in his mathematical abilities than we recalled at the beginning of the program. Now, at the end of this program, we feel that this student is now ready to be challenged with more advanced mathematical concepts and skills that would be considered appropriate for a level three student. (Below are charts that document this growth).

Student 2: ESGI Assessment Data

	FALL	**WINTER**	**SPRING**
Number Recognition	5 out of 32 correct	16 out of 32 correct	21 out of 32 correct
Rote Counting	1 out of 10 correct	3 out of 10 correct	4 out of 10 correct
Comparing Numbers	4 out of 8 correct	7 out of 8 correct	8 out of 8 correct
Recognizing Shapes	2 out of 18 correct	4 out of 18 correct	7 out of 18 correct
Counting Objects	N/A	32 out of 31 counted[2]	N/A
Writing Numbers	1 out of 30 correct	14 out of 30 correct	1 out of 30 correct

[2] Student 2 counted an object twice. Therefore, the student said there were 32 blocks rather than 31 blocks.

Student 2: ESGI Assessment - Fall

Correct	Incorrect
1, 2, 5, 7, 9	0, 3, 4, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31

Correct	Incorrect
10	20, 30, 40, 50, 60, 70, 80, 90, 100

RI- Compare Numbers Q1
(4 out of 8)

Correct	Incorrect
1 3, 10 8, 3 2, 5 2	4 5, 6 9, 7 3, 8 10

Opt. K Qtr. 1 Shapes
(2 out of 18)

Correct	Incorrect
Circle, Triangle	Cone, Cube, Cylinder, Diamond, Heart, Hexagon, Moon/Crescent, Octagon, Oval, Pentagon, Rectangle, Semi-Circle, Sphere, Square, Star, Trapezoid

Student 2: ESGI Assessment - Winter

RI- Numbers Q2
(16 out of 32)

Correct	Incorrect
0, 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 26, 27, 28	9, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 29, 30, 31

RI- Counting Q2
(3 out of 10)

Correct	Incorrect
10, 20, 30	40, 50, 60, 70, 80, 90, 100

RI- Compare Numbers Q2
(7 out of 8)

Correct	Incorrect
1 3, 10 8, 3 2, 4 5, 5 2, 7 3, 8 10	6 9

Quarter 2 Shapes
(4 out of 18)

Correct	Incorrect
Circle, Cone, Square, Triangle	Cube, Cylinder, Diamond/Rhombus, Heart, Hexagon, Moon/Crescent, Octagon, Oval, Pentagon, Rectangle, Semi-Circle, Sphere, Star, Trapezoid

Student 2: ESGI Assessment - Spring

RI- Numbers Q3
(21 out of 32)

Correct	Incorrect
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22	20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31

RI- Counting Q3
(4 out of 10)

Correct	Incorrect
10, 20, 30, 40	50, 60, 70, 80, 90

RI- Compare Numbers Q3
(8 out of 8)

Correct	Incorrect
10 8, 3 2, 4 5, 5 3, 5 2, 6 9, 7 3, 8 10	

Ouarter 3 Shapes
(7 out of 18)

Correct	Incorrect
circle, cone, crescent/moon, heart, square, star, triangle	cube, cylinder, hexagon, octagon, oval, pentagon, rectangle, rhombus/diamond, semi-circle, sphere, trapezoid

Students at level three use their previously developed mathematical skills and apply it to higher level thinking skills, such as problem solving. Students at this level develop a repertoire of problem solving strategies and processes while exploring addition, subtraction, number families, and word problems. These students should be provided with a variety of teaching tools in which they can explore and experiment with to help them solve problems. Though students can be taught how to use a tool when first exposed to it, it is essential that they are allowed the time and opportunity to freely explore its use or purpose on their own. Teachers must simply provide the situation for these teaching tools to be used; however it is up to the student to decide how the tool should be used. Teachers should also allow the students to solve the problems in a way that is best for them. For example, one student may prefer to use his fingers while another student may rely on counting chips or other manipulatives. Once a student has used their own method or strategy for solving, they must learn to evaluate their results to determine how they came about their answer and the reasoning that supports their correct answer. This stage or level of number sense focuses greatly on the student's thinking process. Students must be aware of their metacognition and "think about their thinking" to determine which strategies worked and which did not. Though we consider applying number sense skills to problem solving situations to be our third level, we are aware that problem solving skills and capabilities extend way beyond that. Problem solving is central to all mathematics and it can even be applied to critical and higher level thinking and real world situations.

Based on our observations and analysis of his assessment data, it was clear that when we first began working with student three, he had already developed a strong understanding for the number sense skills addressed in levels one and two. Though this student had not fully mastered all skills, we felt confident that he was well prepared to begin exploring and developing higher level thinking skills, such as problem solving. Similarly to our other focus students, student three also explored the targeted skills that were appropriate for his level through both manipulatives and technology. Our goal for this student was to help him develop multiple problem solving strategies. This student explored iPad apps such as *Domino Add, Add Sub, 10 Bead Math* and *Balance Math.* Level three students also used manipulatives such as tangible dominos, a homemade Rekenrek, balance beam, counting chips, etc. Like we noted in our description of student two, student three also helped us understand the importance of allowing students an adequate amount of time to explore a tool before expecting them to use it in demonstrating their understanding. In our first several lessons, this student relied on using his fingers to do simple addition problems. When he was given a teaching tool that he was unfamiliar with, he resorted to using this preferred problem solving strategy. Many of the equations we asked him to solve had sums that were greater than ten. Thus, because this student was using his fingers to solve the problems, this student struggled to complete these equations when he "ran out" of fingers. As time went on, we no longer had to remind this student to use the provided teaching tool. Rather, as the student became more comfortable with the tools, he automatically used them to solve the problems presented. This student became more aware of the problem solving strategies that work best for him as well as how to appropriately use them. We are confident that because this student has developed a repertoire of problem solving strategies that he is now familiar and comfortable with, he will continue to understand how to select the most appropriate strategies in future problem solving situations.

Student 3: ESGI Assessment Data

	FALL	WINTER	SPRING
Number Recognition	14 out of 32 correct	21 out of 32 correct	28 out of 32 correct
Rote Counting	3 out of 10 correct	3 out of 10 correct	6 out of 10 correct
Comparing Numbers	8 out of 8 correct	8 out of 8 correct	8 out of 8 correct
Recognizing Shapes	9 out of 18 correct	8 out of 18 correct	14 out of 18 correct
Counting Objects	N/A	31 out of 31 correct	N/A
Writing Numbers	4 out of 30 correct	13 out of 30 correct	19 out of 30 correct

Student 3: ESGI Assessment - Fall

RI- Numbers Q1
(14 out of 32)

Correct	Incorrect
0, 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 20, 22	9, 13, 14, 15, 16, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31

RI- Counting Q1
(3 out of 10)

Correct	Incorrect
10, 20, 30	40, 50, 60, 70, 80, 90, 100

RI- Compare Numbers Q1
(8 out of 8)

Correct	Incorrect
1 3, 10 8, 3 2, 4 5, 5 2 , 6 9, 7 3, 8 10	

Opt. K Qtr. 1 Shapes
(9 out of 18)

Correct	Incorrect
Heart, Moon/Crescent, Octagon, Oval, Rectangle, Square, Star, Trapezoid, Triangle	Circle, Cone, Cube, Cylinder, Diamond, Hexagon, Pentagon, Semi-Circle, Sphere

Student 3: ESGI Assessment - Winter

RI- Numbers Q2
(21 out of 32)

Correct	Incorrect
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 22	15, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31

RI- Counting Q2
(3 out of 10)

Correct	Incorrect
10, 20, 30	40, 50, 60, 70, 80, 90, 100

RI- Compare Numbers Q2
(8 out of 8)

Correct	Incorrect
1 3, 10 8, 3 2, 4 5, 5 2, 6 9, 7 3, 8 10	

Quarter 2 Shapes
(8 out of 18)

Correct	Incorrect
Circle, Diamond/Rhombus, Heart, Oval, Rectangle, Square, Star, Triangle	Cone, Cube, Cylinder, Hexagon, Moon/Crescent, Octagon, Pentagon, Semi-Circle, Sphere, Trapezoid

Student 3: ESGI Assessment – Spring

RI- Numbers Q3
(28 out of 32)

Correct	Incorrect
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 29	19, 28, 30, 31

RI- Counting Q3
(6 out of 10)

Correct	Incorrect
10, 20, 30, 40, 80, 90	50, 60, 70, 100

RI- Compare Numbers Q3
(8 out of 8)

Correct	Incorrect
10 8, 3 2, 4 5, 5 3, 5 2, 6 9, 7 3, 8 10	

Ouarter 3 Shapes
(14 out of 18)

Correct	Incorrect
circle, cone, crescent/moon, cube, heart, hexagon, octagon, oval, rhombus/diamond, sphere, square, star, trapezoid, triangle	cylinder, pentagon, rectangle, semi-circle

Conclusion:

After reflecting on our research as well as our experiences throughout the Number Sense Program, it became increasingly clear how much we learned about childhood numeracy and how much we grew as teachers. Through our research we learned of the wide variety of teaching tools that could be made available to students. Prior to this program, we ourselves had limited exposure to the variety of available teaching tools. However, as a result of this program, we are now more aware of these tools and feel as if we will be better equipped for the future. Though these tools have the potential to enhance student learning, it is essential they are used in the appropriate fashion. Teaching tools should be implemented based on the concept or skill being addressed as well as the situation in which they are to be used. We have learned that some teaching tools more appropriately address some concepts and skills than others. We have also learned that once choosing the most appropriate tool to be used, teachers must allow students an adequate amount of time to experiment and explore the tool before expecting them to demonstrate an understanding through its use. Though there are a variety of beneficial teaching tools, it is essential teachers first explore the tools themselves and develop an understanding for its use. They must realize that these tools are "only right when used right" before implementing them into their lessons. Looking back on the past several months, it is obvious that we have grown as teachers. Though we struggled at first, we have now come to understand how to appropriately implement a variety of teaching tools. We know which tools best address each concept and skill, as well as how to best assist our students as they use them. We also feel we are able to plan and deliver more effective lessons that provide us with more valid assessment data. This data, as well as our lesson observations, has allowed us to pinpoint our student's needs. Over the course of this program, we have learned just how to accommodate and meet the individual needs of each learner. We are extremely grateful for this experience and feel that we have truly impacted our students' numeracy development. Though we are able to recognize this growth in our teaching, we know it would not be possible without the many people who

have assisted us throughout this process. We are extremely thankful that Dr. Egan, Dr. Hengst, Ms. Carmack, Mrs. Arnold, and Mrs. Fields took the time to meet with us and give us helpful advice and guidance as we worked towards bettering our professional development as perspective teachers.

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