The Greeks

- The Greeks are used in finance to measure the sensitivity of the price of derivatives (options) to various parameters.
- The Greeks used in the Black-Scholes equation: Delta, Gamma, Vega, Theta, and Rho
- Delta, vega, Rho and Theta are first order, Gamma is second order.
- Other “Greeks” include: Lambda, Vanna, Vomma, Charm, Veta, Vera, Color, Speed, Ultima, and Zomma
Second order Greeks

- **Lambda** – Leverage, % change in option value per % change in the stock price:

\[
\frac{V}{S} \times \frac{S}{V}
\]

- **Vanna** – Sensitivity of the delta of an option with respect to volatility:

\[
\frac{\partial^2 V}{\partial S^2}
\]

- **Vomma** – Sensitivity of vega with respect to volatility:

\[
\frac{\partial^2 V}{\partial \sigma^2}
\]

- **Charm** – Sensitivity of theta with respect to stock price:

\[
\frac{\partial^2 V}{\partial S \partial t}
\]

- **Veta** – Sensitivity of vega with respect to theta:

\[
\frac{\partial^2 V}{\partial \sigma \partial \theta}
\]

- **Vera** – Sensitivity of Rho with respect to volatility:

\[
\frac{\partial^2 V}{\partial \rho}
\]
Third order Greeks

- **Color** – used in monitoring gamma-hedged portfolios
  \[ \frac{\partial^3 V}{\partial S^2 \partial t} \]

- **Speed** – used in monitoring delta-hedged portfolios
  \[ \frac{\partial^3 V}{\partial S^3} \]

- **Ultima** – Measures sensitivity of vomma
  \[ \frac{\partial^3 V}{\partial S^2 \partial \sigma} \]

- **Zomma** – useful for monitoring gamma-hedged portfolio
  \[ \frac{\partial^3 V}{\partial S^2} \]
The Delta

- The delta shows how much a $1 change in the price of the underlying asset will affect the price of an option.
- Suppose a call option has a delta of .40
- A $1 increase in the stock price would increase the call value by $.40
- The delta of the put option would be .40-1.00 = -.60
- This means a $1 increase in the stock price would decrease the put option value by $.60
Delta Hedge

- Attain a Delta Neutral Position (delta = 0)
  - You could use the purchase of stock to offset the selling of a call option
  - Say you sell a call option of 100 shares of a stock priced at $100 and the delta is .40
  - You should then purchase 40 shares of stock
  - If the stock price rises $1, you lose $40 on the call but you gain $40 on the stock
 Gamma

- A measure of an options sensitivity with respect to delta - rate of change of delta for a $1 change in the stock price
- Suppose a gamma of .01, and a delta of .40 on a call option
- If the stock price rises $1 we would expect the delta to increase to .41
- At-the-money options (delta is about .50) will have a larger gamma
- Gamma will decrease as an option moves into the money or out of the money
- Always positive
Rho measures the options sensitivity to changes in the risk-free interest rate.

If the value of Rho is 1.2 and the risk-free interest rate increases by a generous 1%, you would expect the value of the option to increase by %1.20.

Interest rates don’t change often, this makes Rho somewhat insignificant as a sensitivity measure.
Theta

- Sometimes called the time decay on the value of an option
- Refers to how much an option value will increase/decrease per day with respect to time
- Theta is usually negative since the value tends to decrease over time
- Not as significant as other Greeks pertaining to option value
- A theta of -1.2 would mean that the value of the option will decrease by $1.20 a day
Vega

- Vega is a measure of an option’s sensitivity to a change in the volatility of the underlying asset.
- The only non-Greek “Greek”
- Suppose a call option with a stock price $50 and strike price $50, the volatility is .28
- The option costs $5.00 and the vega is 0.15
- In theory, the option value should increase by $0.15 for every .01 increase in volatility.
Delta Equation - Python

- $S =$ stock price
- $K =$ strike price
- $\Sigma =$ volatility
- $r =$ risk-free interest rate
- $\tau =$ time to expiry
- $d_1 = \frac{\log(S/K) + (r + \Sigma^2/2)\tau}{\Sigma\sqrt{\tau}}$
- Plug $d_1$ into the cumulative distribution function:
  - Call Delta $= \frac{1}{2}(1 + \text{erf}(d_1))$
  - Call this $N(d_1)$
  - Put Delta $= N(d_1) - 1$
Gamma, Rho and Vega

- Call Gamma = \( \frac{N(d_1)}{S \sigma \sqrt{\tau}} \)
- The gamma of the put is the same as the gamma of the call
- Call Rho = \( K \tau \exp(-r \tau) N(d_2)/100 \)
- Put Rho = \( -K \tau \exp(-r \tau) N(-d_2)/100 \)
- Where \( d_2 = d_1 - \sigma \sqrt{\tau} \)
- We divide by 100 because Rho is taken with respect to the risk-free interest rate
- Call Vega = \( S N(d_1) \sqrt{\tau} \)
- Put Vega = \( K \exp(-r \tau) N(d_2) \sqrt{\tau} \)
Call Theta = \[(S*sigma)/2*sqrt(tau)]*N'(d1) – r*K*exp(-r*tau)*N(d2)/365

Where N'(d1) is equal to the pdf of d1:

\[ N'(d1) = \frac{1}{sqrt(2*pi)}*exp(-d1**2/2) \]

Put = -S*sigma*N'(d1)*.5*sqrt(tau) + (r*K*exp(-r*tau))*N(-d2)/365

Divide by 365 days since Theta is measured daily


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