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Statistics in *League of Legends*: Analyzing Runes for Last-Hitting

The online gaming industry continues to grow as the millennial generation pours money into whatever is currently at the forefront of gaming technology. *League of Legends*, an M.O.B.A (massive online battle arena) by Riot Games, currently takes up 22% of the PC gaming market. Riot Games keeps such a large share of the market by consistently putting out new content and updating current content in the game based off of user feedback and information found by Riot's Data Analysts. There are innumerable aspects of *League of Legends* that could be individually analyzed. Analysts also can put testing into items bought in-game and runes bought out-of-game to see if they are balanced correctly or priced correctly. In the game *League of Legends*, by Riot Games there are out of game items called 'runes' that give you bonuses to your character during games. Runes are out-of-game items that can be permanently purchased that will give your character in-game bonuses. In *League of Legends* there are three tiers of runes, going from one to three where third tier runes give the largest bonuses and first tier runes give the smallest bonuses. The point of my research is to identify how much of a bonus the different tiers of runes give compared to each other. However instead of just looking at the raw statistics that they give I am looking at how much those statistics improve a player's skill in game through the use of simulating the laning phase, which is approximately the first 12 levels where a character stays in the same lane on the map. I will be using a marksman character, Caitlyn, for my simulations and relying on almost entirely auto attacks, a standard attack that all characters have but are the main focus of marksmen. By doing this we can look at the improvement tier three runes give to a players ability to last-hit minions (landing the killing blow) and at how far behind (time-wise) in an item path you will be without tier three runes.

Introduction to Runes

When a player loads the *League of Legends* client they open a screen they have the option to play a game, go to the shop, or view your or another's profile to see: games won, region ranking, or rune pages. In the shop one can purchase characters to play in game or purchase runes using influence points (ip for short), a currency to spend in the out of game shop that is earned by playing games of *League of Legends*. It can take a lot of time to earn enough ip and some players would rather spend their ip on champions instead of runes. By determining how much of an in-game benefit that these runes give I can help inform players on how much better they can be by using runes.

To put some perspective to why players do not buy runes and why there are three tiers of runes I will put some numbers to how much ip it takes to buy a character or a set of runes and how long it takes to get that much ip. Some information that should be known first: when selecting runes to use for a game you get a page that has 30 slots, 9 for marks (offensive type), 9 for glyphs (magic/utility type), 9 for seals (defensive type), and 3 quintessences (the most powerful and can be of any type). Tier three runes (the strongest runes) cost 205 ip per runes for the marks, seals, and glyphs while the quintessences cost 1025 ip each. This is a total of 8,610 ip for a single rune page, and most people will use between three and eight different rune pages, but we will just look at one of the most popular ones used on marksman.

Time Needed to Acquire Runes

These numbers are meaningless without perspective, so to add perspective we can calculate how many games it will take to be able to buy a set runes. The average match of *League of Legends* will last around 35 minutes and the average player will typically play around

three games a day winning half of them (loltimeplayed.com). The average ip per win is 81 while a loss earns around 49 ip. There is also an additional 150 bonus ip for your first win of the day, however the bonus can only be applied every 22 hours. So over two days and six games you earn an average of 690 ip for 210 minutes of gameplay, which can be broken down into an average fo 3.29 ip per minute of gameplay. This means it would take 2,620.43 minutes (43.67 hours) of gameplay, an equivalent of 75 games, to get enough ip for one set of tier three runes.

Since new players do not want to spend all of their ip on runes, many will not buy them. Instead they buy more characters to play, and there are over 120 different characters, ranging in price from 450 ip to 6300 ip. Often runes are not purchased until there are no more characters that players really want to buy. Seeing this might be a problem Riot Games created significantly cheaper runes (tier one and tier two) that give slightly smaller bonuses.

One of the alternatives to having tier three runes is tier two runes. Tier two runes cost significantly less while still giving almost the same bonus. Where the totals benefits or a tier three runes would give a bonus of around 10 damage a tier two rune page would give 8 damage. Comparing this to the statistics of the characters that start with around 63 damage these number do make a difference but a 2 damage difference is not much. The cost of these runes compared is much less though, a tier two rune page averages a cost of 1245 ip, 6.3 hours, 7365 less ip or 37.3 hours of gameplay. A more favorable option for those trying to buy more character to play. The other option is to buy tier one runes, the cheapest and weakest option. These runes cost a total of 180 ip but give only half the stats that the tier three runes give, but only take two games to acquire.

Rune Tier	IP Cost	Hours Played Needed	% of Bonus Compared to Tier 3 Runes
Third (Best)	8,100	43	100%
Second	1,245	6.3	~79.5%
First (Worst)	180	0.9	~50%

Table 1 (Graph of rune tiers, ip cost, and hours of gameplay needed to obtain that tier of runes)

Defining the Bonuses Runes Give

Now that we know how long it can take to acquire the different tiers of runes, the next thing to look at is how much of a bonus these runes give. One way to look at the bonuses given is to look at the raw statistics. One cannot determine how much a player improves from raw statistic boosts, but other methods may be used to see the ways runes improve gameplay. For example, if the runes were converted into an in-game item how much would they cost and how long it would take to get that item. After finding out how long it would take to acquire enough gold to purchase that item in-game we can look at how much longer it will take to purchase the core items in a character's item path. The time it takes to build the full item path between each set of runes can help determine how necessary each tier of runes are to keep up with the pace of the game. The other way is to simulate games and check how much damage and gold differences there are between the different rune tiers by the end of the laning phase.

A gold efficiency converter can be used to calculate how much gold (in-game currency) rune stats are worth. A gold efficiency converter is a process of using a gold efficiency graph to determine the gold value of stats in combination with the gold efficiency chart (see table 2). The basic items in *League of Legends* give only one kind of one stat and then we break it down to how much gold equals one of that stat, i.e. an item that gives 10 damage and cost 360 gold would have a gold efficiency of 36 gold per damage. Using the gold efficiency of different items we can find out how much a set of runes is worth in gold, which is how much an item barring their stats would cost.

Champion Statistic	Base Item	Item Cost	Stat Raised	Gold/Stat
Attack Damage	Long Sword	350	10 AD	35
Ability Power	Amplifying tome	435	20 AP	21.75
Armor	Cloth Armor	300	15 Armor	20
Magic Resistance	Null-Magic Mantle	450	450 MR	18

Table 2 (Part of the gold efficiency chart, only showing 4 of the 18 different stats)

After calculating the gold value we can then run simulations of the different tiers of runes to see how long it would take to get that much gold. For having no runes it we can simply find the time it takes to get the gold value of the tier three runes, as this is the starting set of runes for

most committed *League of Legends* players. However, for the tier two runes we have to first find the difference in in the gold values between tier two and three runes, and then see how long it takes a simulation on tier two runes to acquire that much gold. After that we can find out how long it takes for the rune page to get each major item in their core build and see how far behind having no runes and tier two runes are to tier three runes.

The next way to see the worth of the runes is to simulate games and to see how much difference in damage and gold earned there is between the games with each set of runes applied. To simulate games of *League of Legends* is near impossible because of the unpredictability of the player versus player environment. Instead of trying to simulate and entire game we will be focusing on the last-hitting mechanic of the laning phase. The last-hitting mechanic is most crucial during the laning phase of *League of Legends*, as this is the marksman's main source of income, and it is when the runes are at their most effective. This phase of the game is also the easiest to simulate.

Running Simulations

Many aspects of the laning phase would have to be coded and simulated in python. Among the most important to code correctly was the behavior of the minions and the character being played. I coded the number of minions per wave, the amount of experience and gold given by the different types of minions, and that each third wave had one siege minion in it. For the character I had to choose first what character to simulate, plug in their stats, and pick out what runes to use. The character chosen was Caitlyn, a marksman. She was chosen since she is one of the simplest characters in the game to use the last-hit mechanic with since she does not use her abilities often against minions and she is of the marksman class. I chose a marksman since the

class is the simplest in terms damage and there is low variation between the different marksmen type characters. Since most marksmen play similarly and do not use abilities as often as any other character type simulating one of them will have similar results to simulating other marksmen, so this data found will cover the most characters compare to simulating other character types.

The next part of the simulation is to determine is the order in which items are bought and what runes to use. For the item path I went to the site MobaFire.com a site that has the top trending item builds used by high tier players, and commonly referenced by everyday *League of Legends* players, and picked the top rated item path for Caitlyn to use. The most common marksmen rune pages displayed: 9 marks of physical damage, 9 seals of armor, 9 glyphs of magic resist, and 3 quintessences of attack speed.

Most of the information for making my simulation I was able to simply find by looking at the *League of Legends* web page and their Wikipedia page such as the base damage of Caitlyn, the scaling of her damage per level, the damage of her abilities, the experience needed to level up, the experience and gold given by minion and the scaling for those numbers, etc. However, a statistic that was not public information was how likely a player was to last-hit a minion using the different rune tiers since it varies from player to player. To find these numbers I played custom games with each tier of runes and went back and watched the games to find on average how many minions I last-hit (as a percentage). After getting the results I looked at the standard deviation between the trials and found that it was only around 10%, low enough to count my data as accurate. The potential source of error here is that the data was collected from an above average player (top 6% in North America) so some of these number may be different than an average players (leagueofgraphs.com).

When playing the game to collect data I followed the same patterns each time. I bought the starting items that I found in the most used MobaFire builds, walked to the lane that the character normally goes, and last-hit the minions, the mechanic of only hitting the minion after it has lost health and the attack will kill it in one shot. When I had enough gold to buy the next item (or item that eventually builds into the final item) recommended in the MobaFire build I chose, I backed to purchase the item and returned to the lane. The only difference between games was the small changes in the minion pathing and the subtle changes in my gameplay.

Results

After plugging in the numbers and running the simulations of games of *League of Legends* the results ended up being larger than initially thought. Over the course of a laning phase not having runes on contributed to over a 15% decrease in damage dealt compared to tier three runes. With the tier three runes the average damage per game (over 10,000 games) was 69,465 where with no runes on it was 58,783. Tier two runes, though having 80% of the stats of tier three, had an average of 65,702 damage per game, dealing over 5% less damage than tier three runes (see table 3). The average gold earned follows the same pattern but slightly less drastic. For the tier three runes the average gold earned was 8,574, for tier two 8,282, and for no runes 7,666. This is only a max difference of ~10% compared to the 15% for damage dealt (see table 4).

Rune Tier	Average Damage Dealt	Effectiveness Compared to Tier 3
Three	69,465	100%
Two	65,702	94.58%
No Runes	58,783	84.62%

Table 3

Rune Tier	Average Gold Earned	Effectiveness Compared to Tier 3
Three	8,574	100%
Two	8,282	96.59%
No Runes	7,666	89.41%

Table 4

Next I wanted to see how these numbers change if someone were to be perfect at last-hitting, that is they last-hit every minion. For the gold earned all the numbers would be maxed out and there is not anything of special note there. For the damage dealt we see this (see table 5). These differences are much smaller because the program does not have to take into account that

have more damage gives a higher chance to get a last hit, so these numbers are just the proportion of the extra stats the runes give compared to overall attack damage. The runes from tier 3 start off giving a 12% increase to damage compared to no runes (62 damage compared to 70) but as items are bought that percent goes down until you reach the numbers below (table 5) after a few items.

Rune Tier	Average Damage Dealt	Effectiveness Compared to Tier 3
Three	77,083	100%
Two	76,175	98.82%
No Runes	72,674	94.28%

Table 5 (Damage dealt using perfect play)

After finding the differences in damage dealt over a game and gold over a game, the next exploration is how helpful is that gold difference. To find this we can look at what point in a game players have enough gold for the items they buy and we can them compare how much longer it takes to get those items with no runes on. After changing the code slightly the results found seemed minion between tier two and tier three runes but large if no runes were used (see table 6). While the difference in time for buying the first item may be small, over the course of the game the gap widens to over two minutes. For a game that last 35 minute, two minutes can be the difference between winning and losing.

Rune Tier	Time to First Item (B.F. Sword)	Time to Second Item (Infinity Edge)	Time to third Item (Rapid Firecannon)
Three	311 sec	755 sec	1223 sec
Two	321 sec	799 sec	1263 sec
No Runes	346 sec	837 sec	1353 sec

Table 6 (Time needed to get the first three items in the item path)

These numbers represent how far behind in the item path you are, but not how far behind in damage you are. This table does not account for the stats that runes give, so even though it takes 35 more seconds to get the first items without having runes on, even after that item you still have less damage than tier three runes since runes also give a boost. To help account for this time was added to the no runes and tier two runes trials to see how long it would take them to buy an item worth the stats of the tier three runes. For no runes this was easy, all that was done was calculating the gold value of the tier three runes (1045 gold) the code was used to see how long it would take to get that much (see table 2 for the conversion of rune stats to gold). For tier two runes the difference was found between tier three gold cost and tier two gold cost and that time was added instead. The new times can be seen on table 7.

Rune Tier	Time to First Item (B.F. Sword)	Time to Second Item (Infinity Edge)	Time to third Item (Rapid Firecannon)
Three	311 sec	755 sec	1223 sec
Two	370 sec	828 sec	1312 sec
No Runes	611 sec	1102 sec	1618 sec

Table 7 (Time needed to buy items and the stats of tier 3 runes)

The times for the tier two runes only went up slightly (49 sec) but the no runes went up significantly. Now by the end of the third item instead of being two minutes behind the no runes are over six and a half minutes behind. Similarly the tier two runes are now almost one and a half minutes behind now by the third item. Using this as a reference instead of damage dealt or just looking at the stats given by runes shows just how effective runes can be.

Final Thoughts

These results, as shocking as they may be, are not perfectly accurate. Some areas that could have led to some errors in the results. The first source of error was in the low number of trials ran to get the last-hit chance between the different runes. While the standard deviation between the trials was low enough to accept the results, it could be lower by having more trials. The next source of error was the trials were obtained from platinum 3 player (top 6% in North America) so the numbers are not as geared towards the average player. The third source is that in

the code, time is in increments of 5 seconds. They are in 5 second increments because instead of programing time directly into the code, it was made by using minions as a reference point, that is since there are 6 minions per wave and minions spawn every 30 seconds the code said when a minion dies 5 seconds pass (except on waves with 7 minions). Some of these potential error sources may be small but they can add up to make potentially large changes in the numbers, but looking at real games the results seem to be true.

In a fast paced game that lasts 35 minute, if you can get a two minute let alone over a six minute lead on the opponent then that is a significant bonus, Tier 3 runes were shown to give this much of a bonus when a game is allowed to go on this long. Comparing the boost to damage we see what was expected, a small difference between the tier two and tier three runes and a larger difference between having no runes and have tier three. For players that are new to the game I would recommend investing the time to get at least tier two runes, as there are many characters that they will want to buy. For players who want to play ranked and see how high they can get I would strongly encourage them to get tier three runes first, for the bonuses they give matters in less than a minute, but in *League of Legends* a few seconds can turn the tide of a game.

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